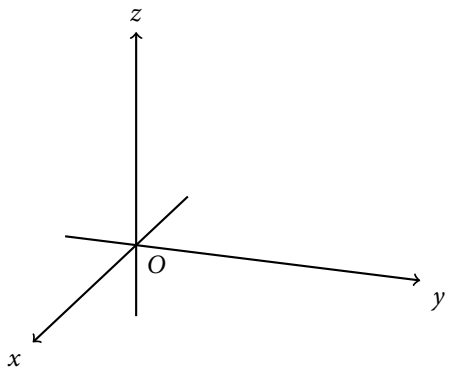
Lesson 1. Three Dimensional Space

1 In this lesson...

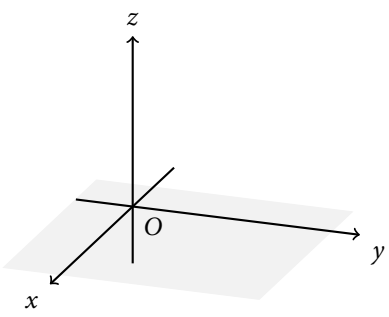
- 3D rectangular coordinate system
- Graphing equations in 3D
- Distance formula in 3D
  - Equation for a sphere

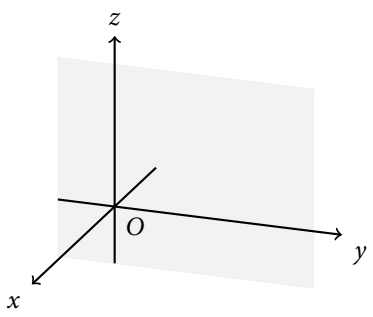
2 3D rectangular coordinate system

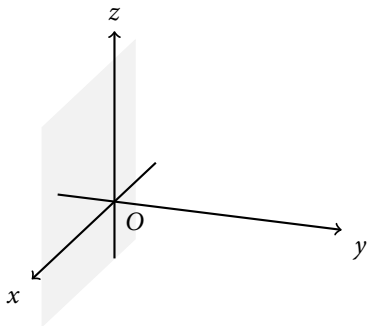
- How do we locate points in *space*?
- 3 mutually perpendicular **coordinate axes** through origin O:



- 3 **coordinate planes**

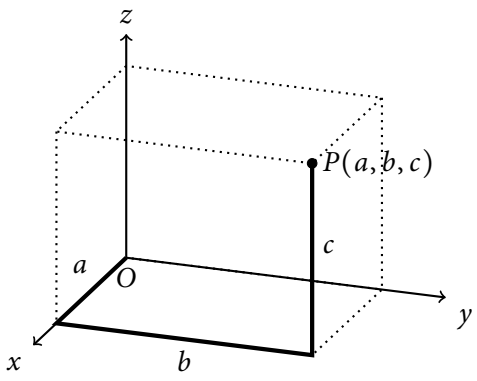






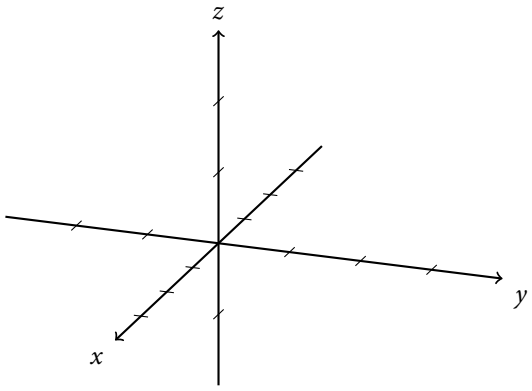
- The coordinate planes divide space into 8 **octants**
  - The **first octant** is the octant with positive axes

- Any point P in space can be represented as an ordered triple (a, b, c):



- (a, b, c) are the **rectangular coordinates** of P (also known as **Cartesian coordinates**)
  - a is called the x-**coordinate** of P
  - b is called the y-**coordinate** of P
  - c is called the z-**coordinate** of P
- Recall we often refer to the two-dimensional plane as $\mathbb{R}^2$
- We often refer to three-dimensional space as $\mathbb{R}^3$

Example 1. Plot $P(3, -2, 2)$.

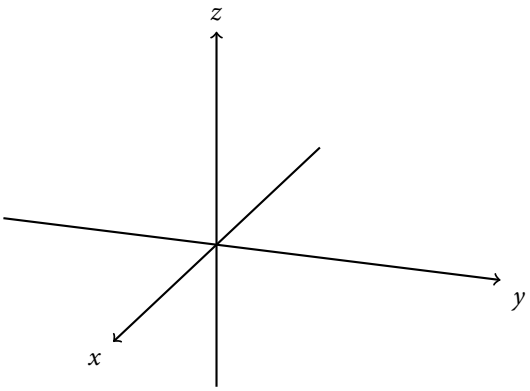


Example 2. Find the distance from $P(3, -2, 2)$ to (a) the xy-plane, and (b) the xz-plane, and (c) the x-axis.

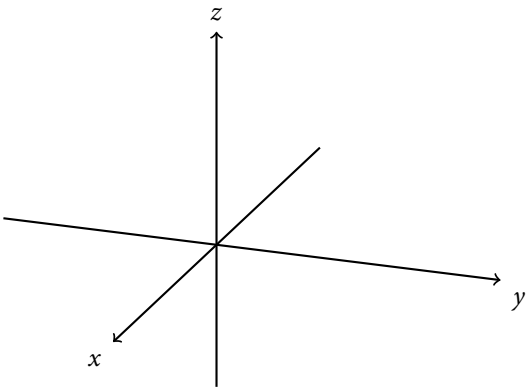
3 Graphing equations in 3D

- Recall that in 2D: the graph of an equation in x and y is a curve in $\mathbb{R}^2$
- In 3D: an equation in x, y, and z is a **surface** in $\mathbb{R}^3$

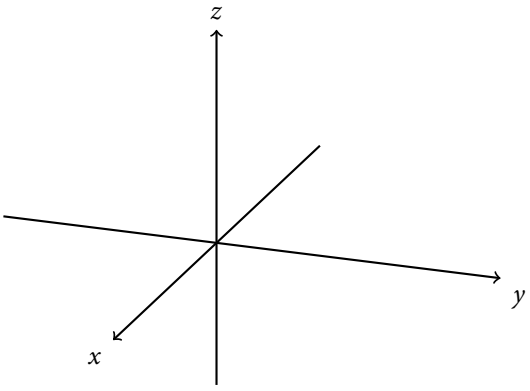
Example 3. Which points satisfy $y = 1$ in $\mathbb{R}^3$?



Example 4. Which points satisfy $y = x^2, z = 0$ in $\mathbb{R}^3$?



Example 5. Which points satisfy $y = x^2$ in $\mathbb{R}^3$?



4 Distance formula in 3D

- Recall the 2D distance formula: the distance between two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ in $\mathbb{R}^2$ is

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- The **distance** between two points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ in $\mathbb{R}^3$ is

Example 6. What is the distance from the point $P(2, -1, 0)$ and $Q(4, 1, 1)$?

- A **sphere** is the set of all points $P(x, y, z)$ whose distance from a center $C(h, k, l)$ is radius r, or

- The standard equation for a sphere with radius r and center (h, k, l) is

Example 7. What region in $\mathbb{R}^3$ is represented by the following inequalities?

$$1 \leq x^2 + y^2 + z^2 \leq 4 \qquad z \leq 0$$